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AN APPROXIMATE SOLUTION OF LONGITUDINAL DISPERSION OF MISCIBLE FLUID FLOW THROUGH POROUS MEDIA

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ABSTRACT

In this paper we have discussed the miscible displacement. This displacement plays important role in research and studying the problem of such displacement of fresh water by sea water in coastal areas for the hydrologist. The problem has become important to people who are trying to dispose safely underground of ever increasing amounts of atomic waste products from nuclear reactors. The miscible displacement has also become useful in oil industries as well as chemical industries.

KEYWORDS: Longitudinal, Dispersion, Miscible, Porous Media, Phenomenon

Mathematics Subjects Classification: 76DXX

1. INTRODUCTION

We are discussing particularly longitudinal dispersion phenomenon which is the process by miscible fluids in laminar flow mix in the direction of the flow. This phenomenon is discussed by observing the cross-sectional flow velocity as time dependent in a specific form. The mathematical formulation of this phenomenon yields a non-linear partial differential equation which is transferred into ordinary differential equation by separation of variables technique. Also, this partial differential equation is solved by the method of two parameters Ritz approximation function. An approximate solution of this problem is obtained in terms of exponential and trigonometric functions.

The hydrodynamic dispersion is the macro scope outcome of the actual movement of individual tracer particles through the pores and various physical and chemical phenomenon simultaneously occurs due to molecular diffusion and convection. Several authors have discussed this same problem in different view points for Carrier [2], Bear [1], Raval, D.[8], Patel, S.S. [5], Patel, D.M. [6], Patel, R.S. [7] etc.

2. FORMULATION OF THE PROBLEM

We are discussing particularly longitudinal dispersion phenomenon which is the process by miscible fluids in laminar flow mix in the direction of the flow. This phenomenon is discussed by observing the cross-sectional flow velocity as time dependent in a specific form. The mathematical formulation of this phenomenon yields a non-linear partial differential equation which is transferred into ordinary differential equation by separation of variables technique. Also, this partial differential equation is solved by the method of two parameter Ritz approximation function. An approximate solution of this problem is obtained in terms of exponential and trigonometric functions. The hydrodynamic dispersion is the macro scope outcome of the actual movement of individual tracer particles through the pores and various physical and chemical phenomenons simultaneously occurs due to molecular diffusion and convection. Several authors have discussed this same problem in different view points for Carrier [2], Bear [1], Raval, D.[8], Patel, S.S. [5], Patel, D.M. [6], Patel, Miscible displacement is nothing but one type of double phase flow in porous media in which the two phases are completely soluble in each other. Thus a capillary force between these two fluids is ineffective. The miscible displacement idea could be described in a very simple form at first Darcy's law, followed by the mixture under condition of complete miscibility, could be thought to check, as a single phase fluid. The change of concentration would be caused by diffusion along the flow channels and thus be governed by the bulk coefficient of diffusion of the fluid in the other. In this form, one comes at a heuristic description of miscible displacement which looks very proper.

The problem is to describe the growth of the mixed region i.e. to find concentration as a function of time t and position X, as the two miscible fluids flow through porous media. Outside of the mixed zone (on either side) the single-fluid equations describe the motion. The problem is more complicated, even in one dimension with fluids of equal properties, since the mixing takes place both longitudinally and transversely. Suppose at t=0, we inject a 'dot' of traced fluid of concentration Co rather than over the entire face. This situation is sketched in the following Fig. The dot moves from left to right it will spread in the direction of flow and perpendicular to the flow. At the right the dot has transformed into an ellipse with concentration varying from C to Co across it.

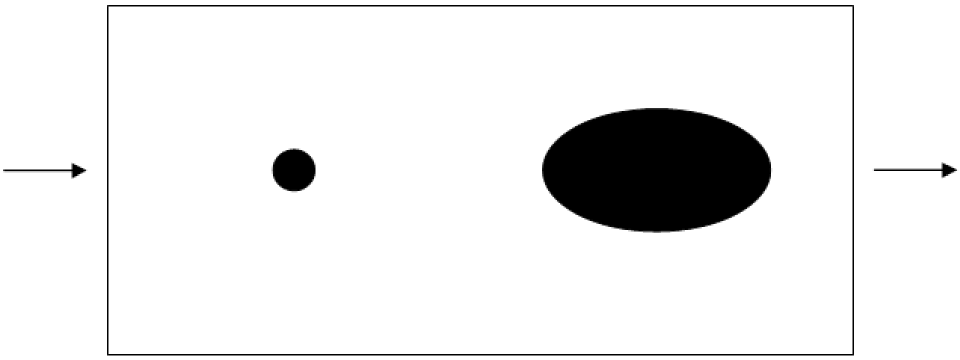

Figure 1: Longitudinal and Transverse Dispersion

According to Darcy's law, the equation of continuity for the mixture, in the case of incompressible fluids, is given by

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \overline{V}) = 0 \tag{2.1}$$

Where ρ is the density for the mixture. V is the pore seepage velocity vector. The equation of diffusion for a fluid flow through homogeneous porous medium, without increasing or decreasing the dispersing material is given by

$$\frac{\partial C}{\partial t} + \operatorname{div}(C\overline{V}) = \operatorname{div}\left[\rho \overline{D} \operatorname{div}\left(\frac{C}{\rho}\right)\right] \tag{2.2}$$

Where C is the concentration of the fluid A in to the other fluid B (Host). (i.e. C is the mass of A per unit volume of the mixture) and D is the tensor coefficient of dispersion with nine components Dij.

In a laminar flow through homogeneous porous medium at constant temperature, ρ is constant. Then

$$\operatorname{div} \bar{V} = 0 \tag{2.3}$$

And equation (2.2) becomes,

$$\frac{\partial C}{\partial t} + \bar{V} \operatorname{div} C = \operatorname{div} (\bar{D} \operatorname{div} C) \tag{2.4}$$

When the seepage velocity V is along the X-axis, the non-zero components are $D_{11} = D_L$ and $D_{22} = D_T$ and other Dij are zero, where D_L is longitudinal dispersion coefficient and D_T is Transverse dispersion coefficient. In this case the equation (2.4) becomes,

$$\frac{\partial C}{\partial t} + \bar{V} \frac{\partial C}{\partial x} = D_L \frac{\partial^2 C}{\partial x^2} \tag{2.5}$$

Where $\bar{V}$ is the component of velocity along the X-axis which is time dependent and $D_L>0$.

Appropriate boundary conditions in longitudinal direction are,

$C (0 , t) = Co \quad ; t > 0$ (2.6)

$C (1 , t) = C_1 \quad ; t > 0$ (2.7)

$C (x , 0) = \epsilon < < 1$ (2.8)

3. SOLUTION BY RITZ APPROXIMATION FUNCTION

Step 1: Multiplying by u the equation (2.5) with $D_L = 1$ and integrating the product over the domain from 0 to 1, the weak form of this equation is given by

$$0 = \int_0^1 u \frac{\partial C}{\partial t} dx + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial C}{\partial x} dx + \bar{V} \int_0^1 u \frac{\partial C}{\partial x} dx - u \left. \frac{\partial c}{\partial x} \right|_{x=0}^{x=1} \tag{3.1}$$

Where u is a test function (an arbitary continuous function).

Step:2 We must select $u = \phi i$; i = 1, 2 in the two-parameter Ritz approximation to satisfy the boundary conditions $\phi i (0) = C_0$, $\phi i (1) = C_1$; i = 1, 2. We choose the following functions as,

$\phi_1 = (C_1 - C_0) \cdot x + C_0$ and $\phi_2 = (C_1 - C_0) \cdot x^2 + C_0$ (3.2)

The Ritz method seeks an approximate solution to equation (2.5) in the form of a finite series

$C (x, t) = b_1 (t) \phi_1 (x) + b_2 (t) \phi_2 (x)$ (3.3)

Where the constants bj ; j = 1,2 called Ritz coefficients are chosen such that equation (3.1) holds for $u = \phi i$; i = 1,2.

Step 3: Substituting the values from equation (3.3) with u = ϕi ; i = 1,2 in the equation (3.1). We get Ritz equation as,

$$b_1' A_{11} + b_2' A_{12} + b_1 B_{11} + b_2 B_{12} + V b_1 E_{11} + V b_2 E_{12} - \phi_1 (b_1 \phi_1' + b_2 \phi_2') \Big|_{x=0}^{x=1} = 0 \tag{3.4}$$

and

$$b_1' A_{21} + b_2' A_{22} + b_1 B_{21} + b_2 B_{22} + V b_1 E_{21} + V b_2 E_{22} - \phi_2 (b_1 \phi_1' + b_2 \phi_2') \Big|_{x=0}^{x=1} = 0 \tag{3.5}$$

where

$$b_1' = \frac{\partial b_1}{\partial t}; \qquad b_2' = \frac{\partial b_2}{\partial t}$$

$$\phi_1' = \frac{\partial \phi_1}{\partial x}; \qquad \phi_2' = \frac{\partial \phi_2}{\partial x}$$

$$Aij = \int_0^1 \phi i \cdot \phi j \, dx ; \quad i, j = 1, 2$$

$$Aij = \int_0^1 \phi i \cdot \phi j \, dx ; i, j = 1, 2 \tag{3.6}$$

$$Bij = \int_0^1 \phi i' \cdot \phi j' \, dx ; i, j = 1, 2 \tag{3.7}$$

$$Eij = \int_0^1 \phi i \cdot \phi j' \, dx ; i, j = 1, 2 \tag{3.8}$$

Step 4: Simplification of equation (3.4) and (3.5) with the help of Aij, Bij and Eij ; i, j = 1, 2 is given by

$$b_1'\left(\frac{C_1^2+C_0C_1+C_0^2}{3}\right)+b_2'\left(\frac{3C_1^2+4C_0C_1+5C_0^2}{12}\right)+b_1\frac{(C_1^2-C_0^2)\bar{V}}{2}+b_2\left(C_0^2-C_1^2+\frac{\bar{V}(C_1-C_0)(2C_1+C_0)}{3}\right)=0 \tag{3.9}$$

and

$$b_1'\left(\frac{3C_1^2+4C_0C_1+5C_0^2}{12}\right)+b_2'\left(\frac{3C_1^2+4C_0C_1+8C_0^2}{15}\right)+b_1\left(\frac{\bar{V}(C_1-C_0)(2C_0+C_1)}{3}\right)+b_2\left(\frac{\bar{V}(C_1^2-C_0^2)}{2}+\frac{4C_0^2-2C_0C_1-2C_1^2}{3}\right)=0 \tag{3.10}$$

Step 5: The residual of the approximation in the initial condition is

$$y = C(x,0) - \varepsilon \tag{3.11}$$

Using the Galerkin method, we have

$$\int_0^1 [C(x,0)-\varepsilon]\,\phi i = 0 \; ; i = 1, 2 \tag{3.12}$$

Means

$$\frac{b_1(0)}{3}[C_1^2+C_0C_1+C_0^2]+\frac{b_2(0)}{12}[3C_1^2+4C_0C_1+5C_0^2]-\frac{\varepsilon C_1}{2}-\frac{\varepsilon C_0}{\;}=0 \tag{3.13}$$

$$\frac{b_1(0)}{12}[3C_1^2+4C_0C_1+5C_0^2]+\frac{b_2(0)}{15}[3C_1^2+4C_0C_1+8C_0^2]-\frac{\varepsilon C_1}{3}-\frac{2\varepsilon C_0}{3}=0 \tag{3.14}$$

And

We obtain approximate initial conditions

$$b_1(0) \cong 0.0044 \quad \text{and} \quad b_2(0) \cong -0.0036 \tag{3.15}$$

Step 6: We can solve the ordinary differential equation (3.9) and (3.10) subject to the initial condition (3.15) by exact means. Using Laplace transform method we obtain

$$L\{b_1(t)\} = \frac{0.0004144}{(p+a)^2 + b} \tag{3.16}$$

And

$$L\{b_2(t)\} = \frac{0.0008666}{(p+a)^2 + b} \tag{3.17}$$

where $a = 1.7386676\, V + 3.2835449$

$$b = 2.1861235\, V^2 - 9.989722\, V - 10.781667 \tag{3.18}$$

$bi = f(p)$; $i = 1, 2$ and p is the variable in the Laplace transform.

Inverting (3.16) and (3.17) we get

$$b_1(t) = \frac{0.0004144}{\sqrt{b}}\, e^{-at} \sin(\sqrt{b} \cdot t) \tag{3.19}$$

and

$$b_2(t) = \frac{-0.0008666}{\sqrt{b}}\, V \cdot e^{-at} \sin(\sqrt{b} \cdot t) \tag{3.20}$$

Step:7 Required solution of the problem is given by

$$C(x,t) = \frac{e^{-at} \sin(\sqrt{b} \cdot t)}{\sqrt{b}} \left[0.0004144\,(0.89 \cdot x + 0.01) - 0.0008666 \cdot V\,(0.89\, x^2 + 0.01) \right] \tag{3.21}$$

where a and b is in equation (3.18).

The following values of the various parameters have been considered in the present analysis and for graphical representation

$C_0 = 0.01$, $C_1 = 0.9$, $\varepsilon = 0.001$

4. GRAPHICAL AND NUMERICAL REPRESENTATION

Table 1

x	t = 0.1	t = 0.2	t = 0.3	t = 0.4
0	4.66187E-08	6.72029E-08	7.58406E-08	7.90043E-08
0.1	2.37181E-06	3.41907E-06	3.85853E-06	4.01949E-06
0.2	4.2725E-06	6.15899E-06	6.95061E-06	7.24056E-06
0.3	5.74867E-06	8.28696E-06	9.3521E-06	9.74222E-06

Table 1				
0.4	6.80034E-06	9.80298E-06	1.1063E-05	1.15245E-05
0.5	7.4275E-06	1.07071E-05	1.20833E-05	1.25873E-05
0.6	7.63015E-06	1.09992E-05	1.24129E-05	1.29307E-05
0.7	7.4083E-06	1.06794E-05	1.2052E-05	1.25548E-05
0.8	6.76194E-06	9.74762E-06	1.10005E-05	1.14594E-05
0.9	5.69107E-06	8.20392E-06	9.25838E-06	9.64459E-06
1	4.19569E-06	6.04826E-06	6.82566E-06	7.11039E-06

Hear $\bar{V}$ = 0.4, a = 3.979012 and b = - 14.4278

The graph shows that when x is increasing for t > 0 (constant), V = 0.4, concentration C of the fluid is increasing uniformly between 0 ≤ x ≤ 0.6 and it is decreasing between 0.6 < x ≤ 1 the graph indicates that x and C are linearly dependent Fig. [4.1]

The graph shows that when t is increasing for x > 0 (constant), concentration C of the fluid is increasing positive exponentially between 0 ≤ t ≤ 0.8 and also it is constant between 0.8 < t ≤ 1 for fixed x = 0.9.

This concludes that the concentration C is constant after some time for fix x Figure. [4.2].

(2)

t	**$\bar{V}$ = 0.1**	**$\bar{V}$ = 0.2**	**$\bar{V}$ = 0.3**	**$\bar{V}$ = 0.4**
0	0	0	0	0
0.1	1.96821E-05	1.48737E-05	1.02108E-05	5.69107E-06
0.2	2.95118E-05	2.20161E-05	1.4917E-05	8.20392E-06
0.3	3.43934E-05	2.53937E-05	1.70221E-05	9.25838E-06
0.4	3.67899E-05	2.69384E-05	1.78987E-05	9.64459E-06
0.5	3.79383E-05	2.7591E-05	1.81958E-05	9.72554E-06
0.6	3.84601E-05	2.78098E-05	1.82204E-05	9.66796E-06
0.7	3.86674E-05	2.78181E-05	1.81176E-05	9.54843E-06
0.8	3.87168E-05	2.77244E-05	1.79558E-05	9.40211E-06
0.9	3.8687E-05	2.75818E-05	1.77675E-05	9.24511E-06
1	3.86177E-05	2.74159E-05	1.75679E-05	9.08482E-06
$\bar{V}$	0.1	0.2	0.3	
a	3.457412	3.631278	3.805145	
b	-11.7588	-12.6922	-13.5818	

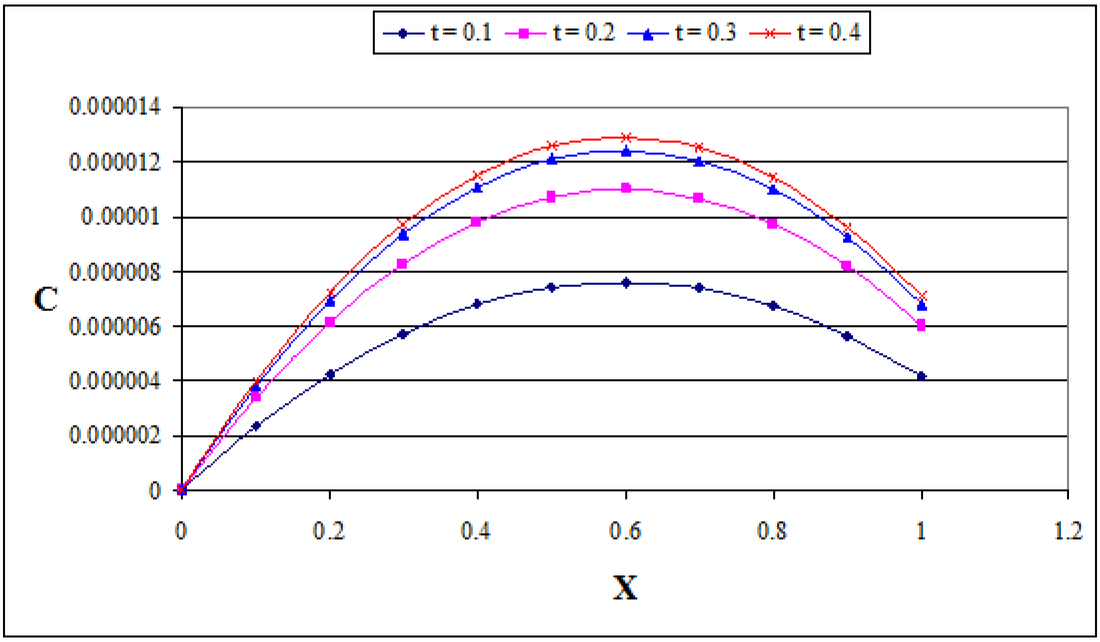

Figure **[4. 1]**

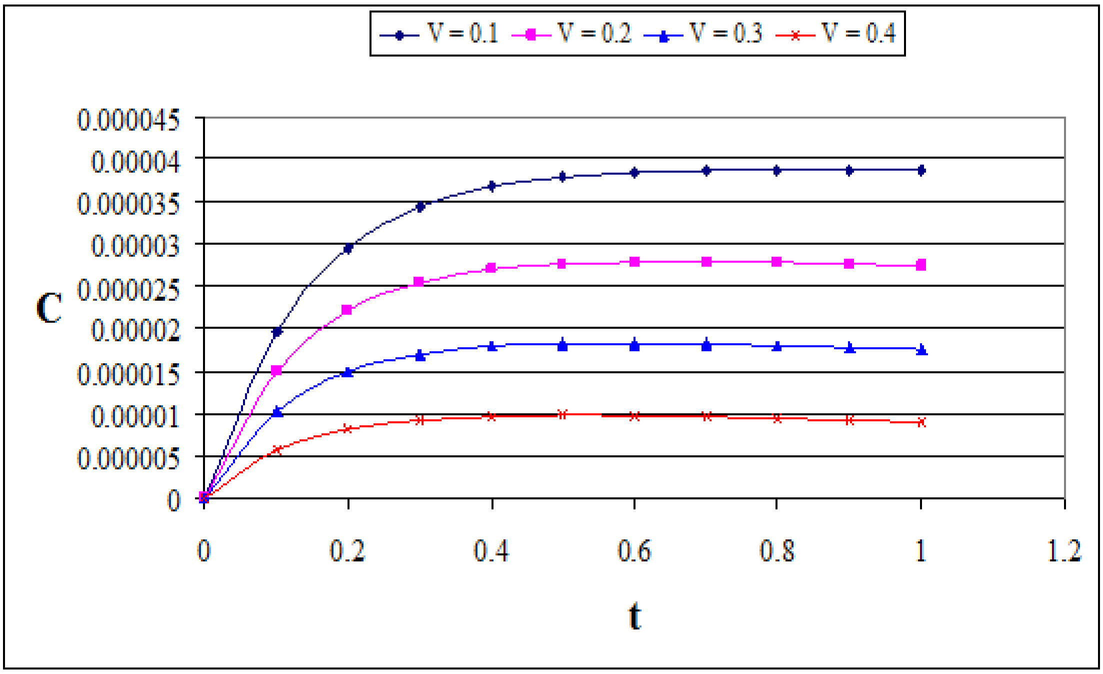

Figure [4.2]

6. CONCLUSIONS

Here, the equation (3.21) represents a solution of a problem of the longitudinal dispersion phenomenon of miscible fluids flow through homogeneous porous media by using some standard and existing assumptions. The solution obtained is approximate and in terms of some terms as well as trigonometric terms by using the method of two parameter Ritz approximation function. Further it is also concluded that the behaviour of the concentration in the longitudinal dispersion is oscillatory at some time of interval for some fix values of x and it become constant after some duration of time. Thus, the result obtained is very feasible with physical problem which opens new direction and interpretation.

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